nag_bessel_y1 (s17adc)

1. Purpose

nag_bessel_y1 (s17adc) returns the value of the Bessel function $Y_1(x)$.

2. Specification

```
#include <nag.h>
#include <nags.h>

double nag_bessel_y1(double x, NagError *fail)
```

3. Description

The function evaluates the Bessel function of the second kind, Y_1, $x > 0$.

The approximation is based on Chebyshev expansions.

For x near zero, $Y_1(x) \simeq -2/\pi x$. This approximation is used when x is sufficiently small for the result to be correct to ***machine precision***. For extremely small x, there is a danger of overflow in calculating $-2/\pi x$ and for such arguments the function will fail.

For very large x, it becomes impossible to provide results with any reasonable accuracy (see Section 6.1), hence the function fails. Such arguments contain insufficient information to determine the phase of oscillation of $Y_1(x)$, only the amplitude, $\sqrt{2/\pi x}$, can be determined and this is returned. The range for which this occurs is roughly related to ***machine precision***; the function will fail if $x \gtrsim 1/$ ***machine precision***.

4. Parameters

x

Input: the argument x of the function.
Constraint: **x**> 0.0.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_REAL_ARG_GT

On entry, **x** must not be greater than ⟨*value*⟩: **x** = ⟨*value*⟩.
x is too large, the function returns the amplitude of the Y_1 oscillation, $\sqrt{2/\pi x}$.

NE_REAL_ARG_LE

On entry, **x** must not be less than or equal to 0.0: **x** = ⟨*value*⟩.
Y_1 is undefined, the function returns zero.

NE_REAL_ARG_TOO_SMALL

On entry, **x** must be greater than ⟨*value*⟩: **x** = ⟨*value*⟩.
x is too close to zero, there is a danger of overflow, the function returns the value of $Y_1(x)$ at the smallest valid argument.

6. Further Comments

6.1. Accuracy

Let δ be the relative error in the argument and E be the absolute error in the result. (Since $Y_1(x)$ oscillates about zero, absolute error and not relative error is significant, except for very small x.)

If δ is somewhat larger than the ***machine precision*** (e.g. if δ is due to data errors etc.), then E and δ are approximately related by: $E \simeq |xY_0(x) - Y_1(x)|\,\delta$ (provided E is also within machine bounds).

However, if δ is of the same order as ***machine precision***, then rounding errors could make E slightly larger than the above relation predicts.

For very small x, absolute error becomes large, but the relative error in the result is of the same order as δ.

For very large x, the above relation ceases to apply. In this region, $Y_1(x) \simeq 2\sin(x - 3\pi/4)/\pi x$. The amplitude $2/\pi x$ can be calculated with reasonable accuracy for all x, but $\sin(x-3\pi/4)$ cannot. If $x - 3\pi/4$ is written as $2N\pi + \theta$ where N is an integer and $0 \leq \theta < 2\pi$, then $\sin(x - 3\pi/4)$ is determined by θ only. If $x > \delta^{-1}$, θ cannot be determined with any accuracy at all. Thus if x is greater than, or of the order of, the inverse of the ***machine precision***, it is impossible to calculate the phase of $Y_1(x)$ and the function must fail.

6.2. References

Abramowitz M and Stegun I A (1968) *Handbook of Mathematical Functions* Dover Publications, New York ch 9 p 358.

Clenshaw C W (1962) *Mathematical Tables, Chebyshev series for mathematical functions* National Physical Laboratory H.M.S.O. **5**.

7. See Also

nag_bessel_y0 (s17acc)

8. Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

8.1. Program Text

```
/* nag_bessel_y1(s17adc) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

main()
{
  double x, y;

  /* Skip heading in data file */
  Vscanf("%*[^\n]");
  Vprintf("s17adc Example Program Results\n");
  Vprintf("     x           y\n");
  while (scanf("%lf", &x) != EOF)
    {
      y = s17adc(x, NAGERR_DEFAULT);
      Vprintf("%12.3e%12.3e\n", x, y);
    }
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
s17adc Example Program Data
             0.5
             1.0
             3.0
             6.0
             8.0
             10.0
             1000.0
```

8.3. Program Results

```
s17adc Example Program Results
     x           y
   5.000e-01  -1.471e+00
   1.000e+00  -7.812e-01
   3.000e+00   3.247e-01
   6.000e+00  -1.750e-01
   8.000e+00  -1.581e-01
   1.000e+01   2.490e-01
   1.000e+03  -2.478e-02
```